Recent results on wheel slip control: Hybrid and continuous algorithms

William Pasillas-Lépine

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Recent results on wheel slip control :
Hybrid and continuous algorithms

William PASILLAS-LÉPINE

Laboratoire des signaux et systèmes

CNRS — Supélec, Gif-sur-Yvette

pasillas@lss.supelec.fr

Collaboration with

Mathieu Gerard (TU Delft), Edwin de Vries (TU Delft), and Antonio Loria (L2S)

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Contents of the talk

- Two main families of ABS algorithms
- Why doing research on ABS today ?
- Other recent approaches
- Continuous wheel slip control algorithms
- Experimental results
- Hybrid five-phase ABS algorithms
- Experimental results
- Conclusions and future work perspectives
- Publications

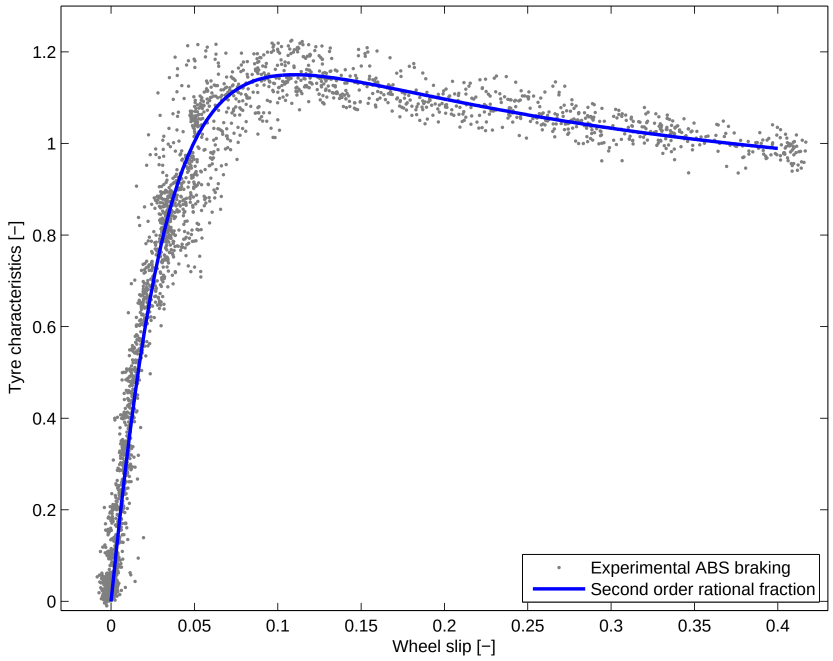
Why do we want to control wheel slip ?

Tyre forces are generated by the wheel slip in the contact patch :

$$\lambda = \frac{R\omega - v_x}{v_x}.$$

They have a nonlinear characteristics with a coupling between longitudinal and lateral forces.

Controlling the wheel-slip improves safety : it reduces the braking distance and maintains steerability.



Two main families of ABS algorithms

Algorithms based on **wheel slip** control :

- it is supposed (implicitly) that vehicle speed is measured (or estimated) ;
- the brake torque converges to a specific value (no oscillations) ;
- mainly present in an *academic* context...
- and in specific applications (ESP, motorcycles, tyre research).

Algorithms based on **angular acceleration** thresholds :

- do not need the vehicle speed, neither the value of optimal wheel slip ;
- quite robust with respect to road conditions and tyre parameters ;
- the brake torque oscillates around the optimal value (limit cycle) ;

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

- mainly present in an *industrial* context ;
- widely diffused on actual vehicles, but completely heuristic.

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Why doing research on ABS today ?

Integrated chassis control :

- black box algorithms are difficult to integrate ;
- open algorithms might clarify the architecture of ICC ;
- decoupling the observation problem (for vehicle speed) from control.

Electric vehicles, In-wheel motors, EMB :

- standard ABS algorithms are not adapted to regenerative braking (Toyota Prius) ;
- these heuristic algorithms need the hydraulic lag in order to work properly...
- they loose performance or do not work at all with electric actuators.

Fault management :

- useful to have algorithms with a stability proof.

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Comparison of our work with other approaches

- We propose a global analysis, not based on linearization — Petersen et al. Nonlinear wheel slip control in ABS brakes using gain scheduled constrained LQR. In *Proc. of the European Control Conference*, 2001.
- Exponential stability in both the stable and unstable tyre domains — Tanelli et al. Robust nonlinear output feedback control for brake by wire control systems. *Automatica*, 2008.
- We take an optimal wheel acceleration setpoint and propose feedforward terms — Savaresi et al. Mixed slip-deceleration control in automotive braking systems. *ASME J. of Dyn. Systems, Measurement, and Control*, 2007.
- Other hybrid approaches that use only wheel acceleration information (Bosch) are based on heuristics, we propose a method based on the analysis of limit cycles.

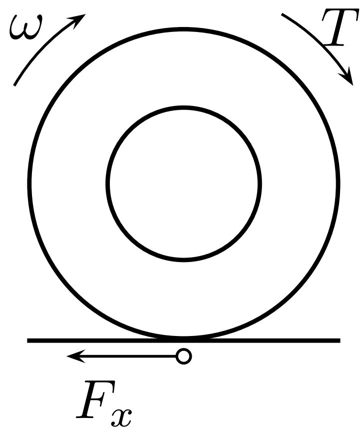
Wheel dynamics

The angular velocity ω of a given wheel of the vehicle has the following dynamics :

$$I\dot{\omega} = -RF_x + T,$$

where I denotes the inertia of the wheel, R its radius, F_x the longitudinal tyre force, and T the torque applied to the wheel.

The torque $T = T_e - T_b$ is composed of the engine torque T_e and the brake torque T_b.



CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Tyre force modelling

The longitudinal tyre force F_x is often modeled as a function

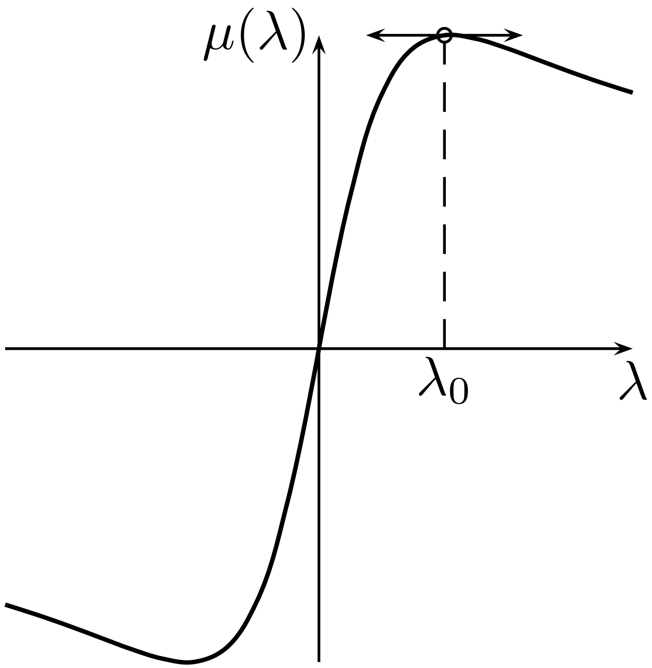
$$F_x = \mu(\lambda) F_z,$$

of the wheel's slip

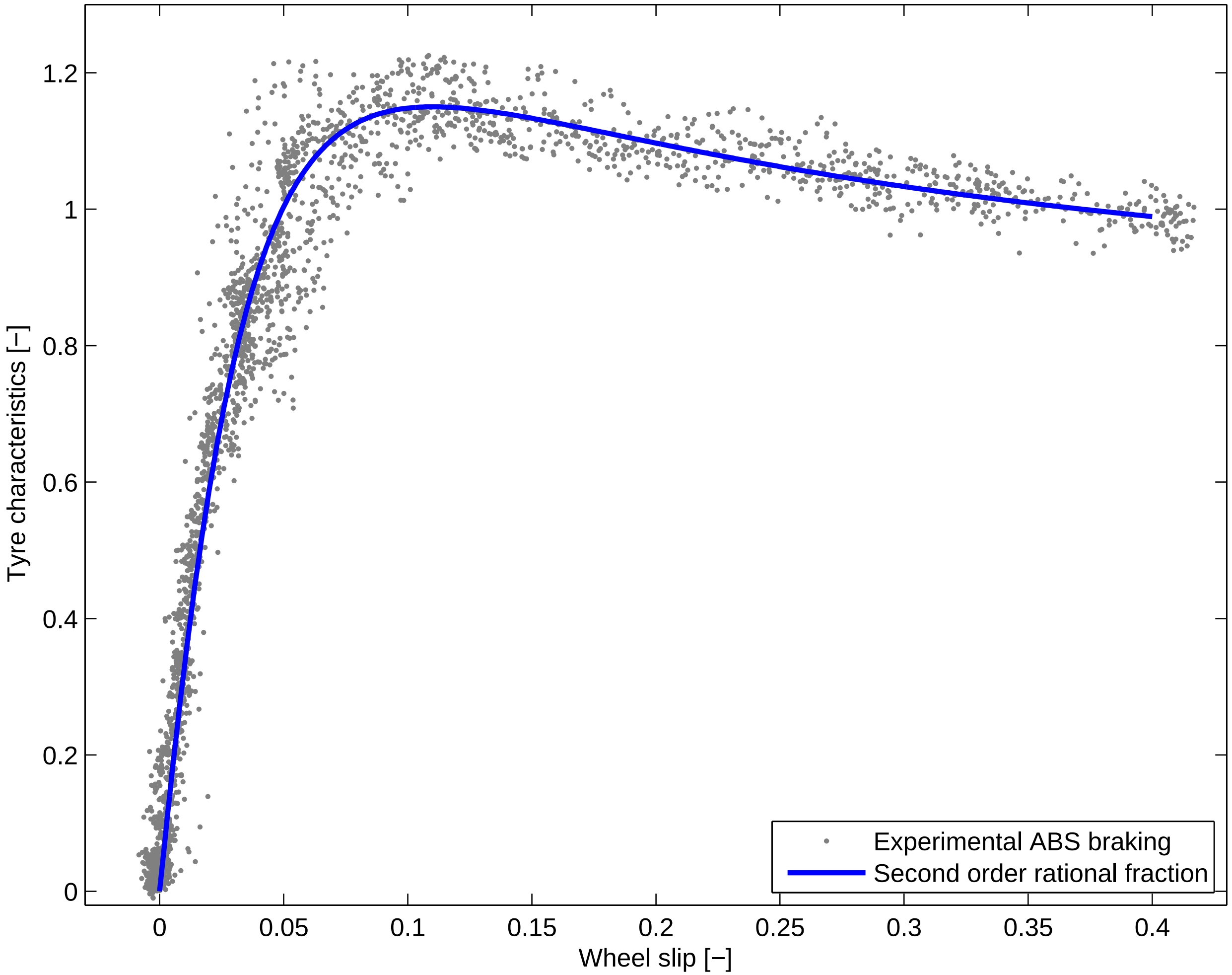
$$\lambda = \frac{R\omega - v_x}{v_x}.$$

The curve $\mu(\cdot)$ can be approximated by a second order rational function

$$\mu(\lambda) = \frac{a_1 \lambda - a_2 \lambda^2}{1 - a_3 \lambda + a_4 \lambda^2}.$$



Experimental validation



Wheel slip and acceleration offsets

Define the variables x_1 and x_2 by

$$\begin{aligned} x_1(t) &= \lambda(t) \\ x_2(t) &= R\frac{d\omega(t)}{dt} - a_x(t), \end{aligned}$$

where $a_x(t)$ is the vehicle's acceleration. Derivating these variables we obtain :

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{1}{v_x(t)}(-a_x(t)x_1 + x_2) \\ \frac{dx_2}{dt} &= -\frac{c\mu'(x_1)}{v_x(t)}(-a_x(t)x_1 + x_2) + \frac{u}{v_x(t)} - \frac{da_x(t)}{dt}, \end{aligned}$$

where

$$c = \frac{R^2}{I}F_z \quad \text{and} \quad u = v_x(t)\frac{R}{I}\frac{dT}{dt}.$$

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Wheel-slip filtered setpoint

For a given wheel-slip reference $\lambda^*(t)$, we will define a filtered setpoint

$$\begin{aligned} \frac{d\lambda_1}{dt} &= \frac{\lambda_2}{v_x(t)} \\ \frac{d\lambda_2}{dt} &= \frac{-\gamma_1(\lambda_1 - \lambda^*) - \gamma_2\lambda_2}{v_x(t)}, \end{aligned}$$

where γ_1 and γ_2 are two positive real numbers.

This setpoint filter gives :

- A smooth reference setpoint (that one can differentiate twice) even if the original setpoint is discontinuous (for exemple, piecewise constant).
- A system for which all equations are divided by the vehicle's velocity. This homogeneity allows an analysis of the system in a new (nonlinear) time-scale in which

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

the dependence on speed disappears.

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Changing the time-scale

In order to have $dt = v_x(t)ds$, we will use a new time-scale

$$s(t) = \int_0^t \frac{d\tau}{v_x(\tau)}.$$

We use a dot to denote the new time-derivative

$$\dot{\varphi}(s) = \frac{d\varphi(s)}{ds}.$$

When the acceleration a_x is constant, in the new time-scale the system is simpler :

$$\begin{aligned}
\dot{x}_1 &= -a_x x_1 + x_2 \\
\dot{x}_2 &= -c\mu'(x_1)(-a_x x_1 + x_2) + u \\
\dot{\lambda}_1 &= \lambda_2 \\
\dot{\lambda}_2 &= -\gamma_1(\lambda_1 - \lambda^*) - \gamma_2\lambda_2.
\end{aligned}$$

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Choice of the operating point

Let $x_1^* = \lambda_1$ be the desired operating point for x_1. Define the error coordinates by

$$\begin{aligned} z_1 &= x_1 - x_1^* \\ z_2 &= x_2 - x_2^*, \end{aligned}$$

where

$$x_2^* = \lambda_2 + a_x x_1 - \alpha z_1 \quad \text{and} \quad \alpha > 0.$$

The closed-loop equation for z_1 reads

$$\dot{z}_1 = -\alpha z_1 + z_2,$$

which is exponentially stable if $z_2 = 0$. The objective is thus to design a control u such that x_2 converges towards x_2^* asymptotically.

Our cascaded control law

Driving x_2 towards the dynamic setpoint

$$x_2^* = a_x x_1 + \lambda_2 - \alpha z_1$$

is achieved using the control law

$$u = \underbrace{-\gamma_1(\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + a\mu'(x_1))\,\lambda_2}_{feedforward} \underbrace{-k_1 z_1 - k_2 z_2}_{feedback}.$$

The dynamic setpoint x_2^* is the core of the cascade :

- The steady state is $a_x x_1$.
- Other terms to reduce error z_1 using cascaded feedback ($-\alpha z_1$) and cascaded feedforward (λ_2).

Global exponential stability

Theorem 1 *Consider an arbitrary piecewise-continuous wheel slip reference $\lambda^*(t)$. If $\lambda^*(t)$ is injected into the filtered setpoint equations and the control law*

$$u = -\gamma_1(\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + c\mu'(x_1))\,\lambda_2 - k_1 z_1 - k_2 z_2$$

is introduced into the system, then a time-varying closed-loop system

$$\dot{z} = \begin{bmatrix} -\alpha & 1 \\ -k_1 + a_x\alpha - \alpha^2 + \alpha\eta(t) & -k_2 + \alpha - a_x - \eta(t) \end{bmatrix} z,$$

is obtained. If the control gains k_1 and k_2 satisfy

$$k_1 > a_x\alpha - \alpha^2 \quad \textit{and} \quad k_2 > \alpha - a_x + \eta_m$$

then the origin of this closed loop system is globally exponentially stable.

Robustness

Corollary 1 *Consider a constant wheel slip reference λ^*. If λ^* is injected into the filtered setpoint equations and the control law*

$$u = -\gamma_1(\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + c\hat{\mu}'(x_1))\,\lambda_2 - k_1 z_1 - k_2 z_2$$

is introduced into the system, then a time-varying closed-loop system

$$\dot{z} = A(t)z + B(t)w \quad \dot{w} = C(t)w$$

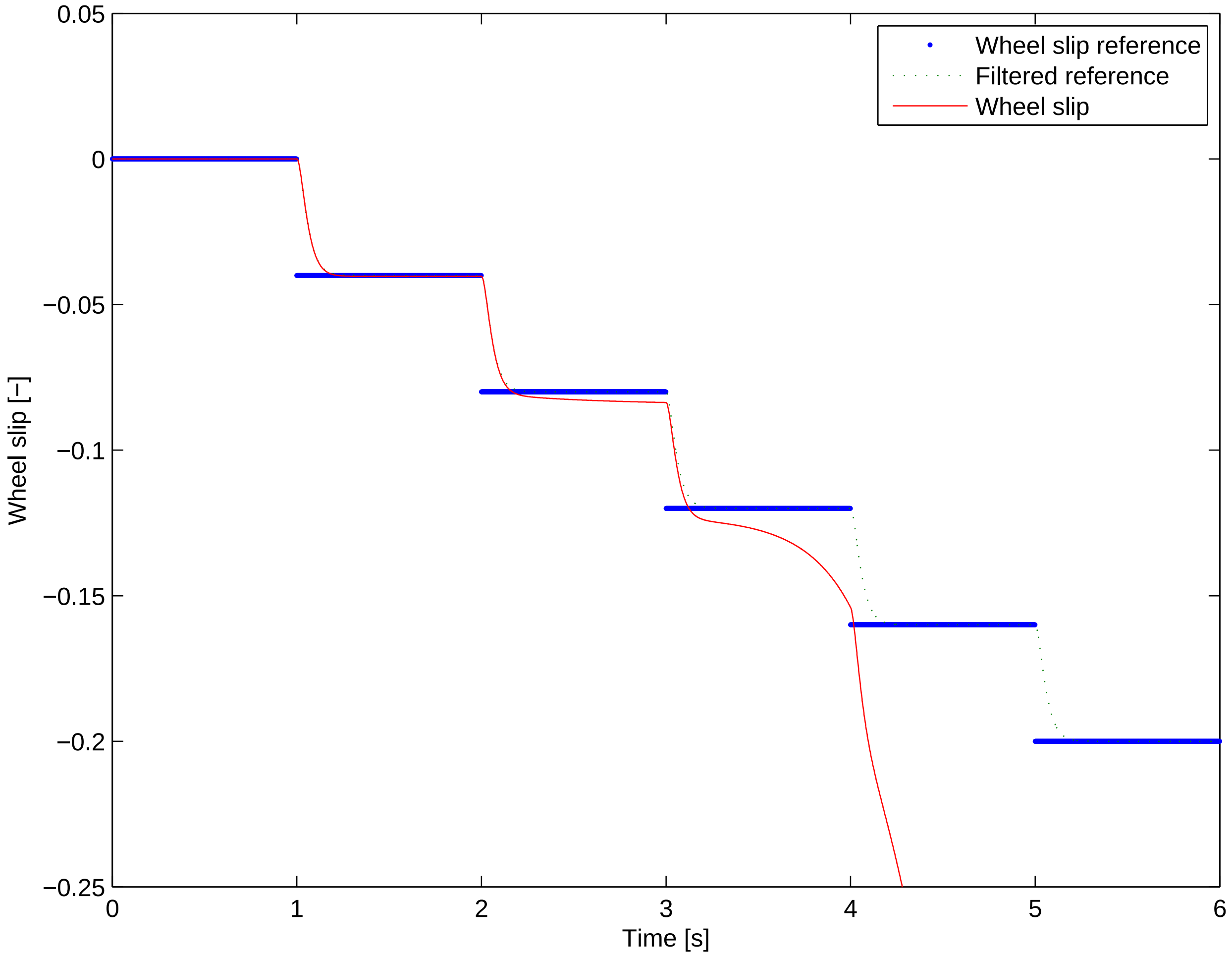
is obtained, with the same matrix $A(t)$ as in Theorem 1, and $w = (\lambda_1 - \lambda^*, \lambda_2)$. If the control gains k_1 and k_2 satisfy the bounds

$$k_1 > a_x\alpha - \alpha^2 \quad \text{and} \quad k_2 > \alpha - a_x + \eta_m$$

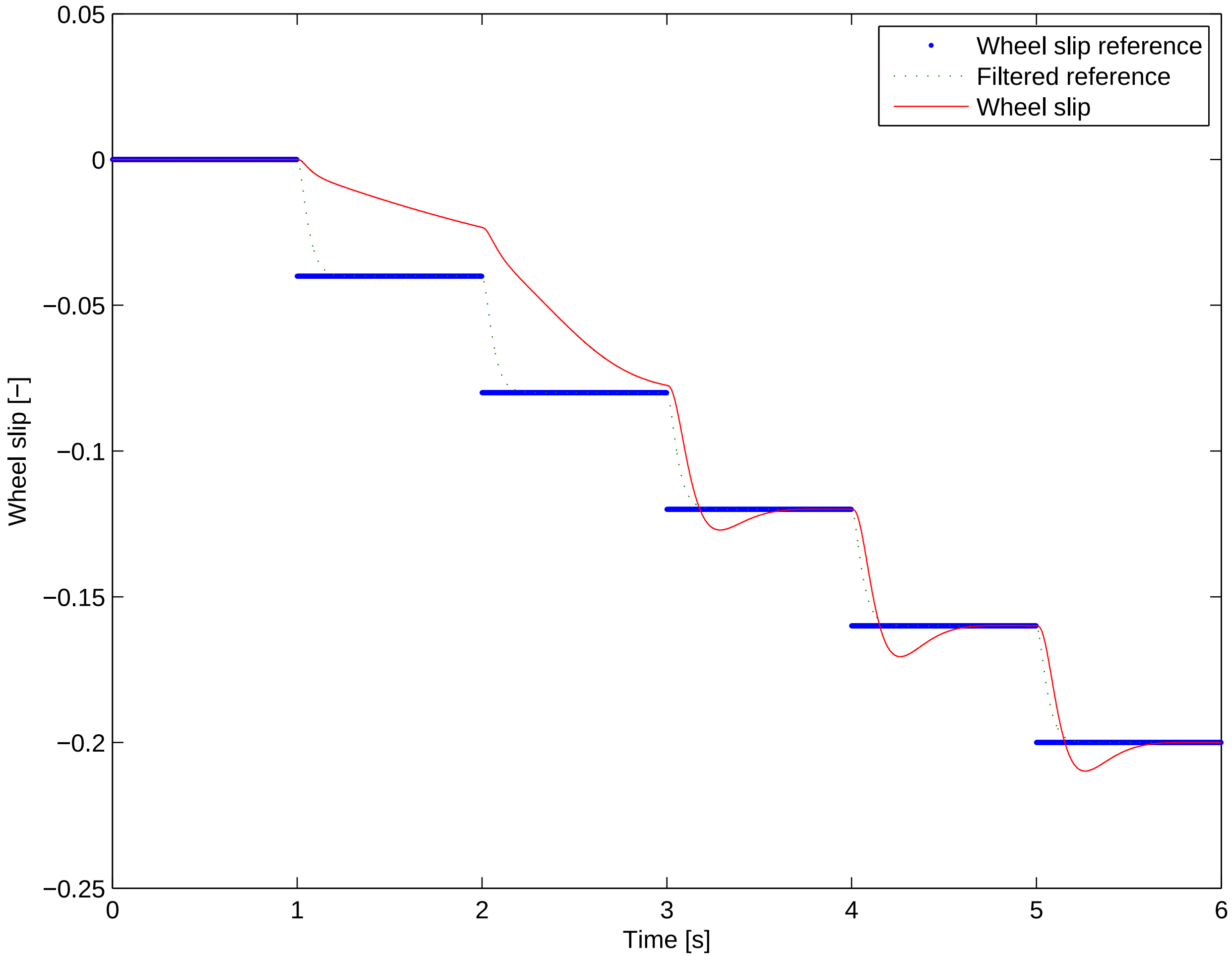
of Theorem 1, then the closed loop system is globally exponentially stable.

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

Simulations — Pure feedforward control

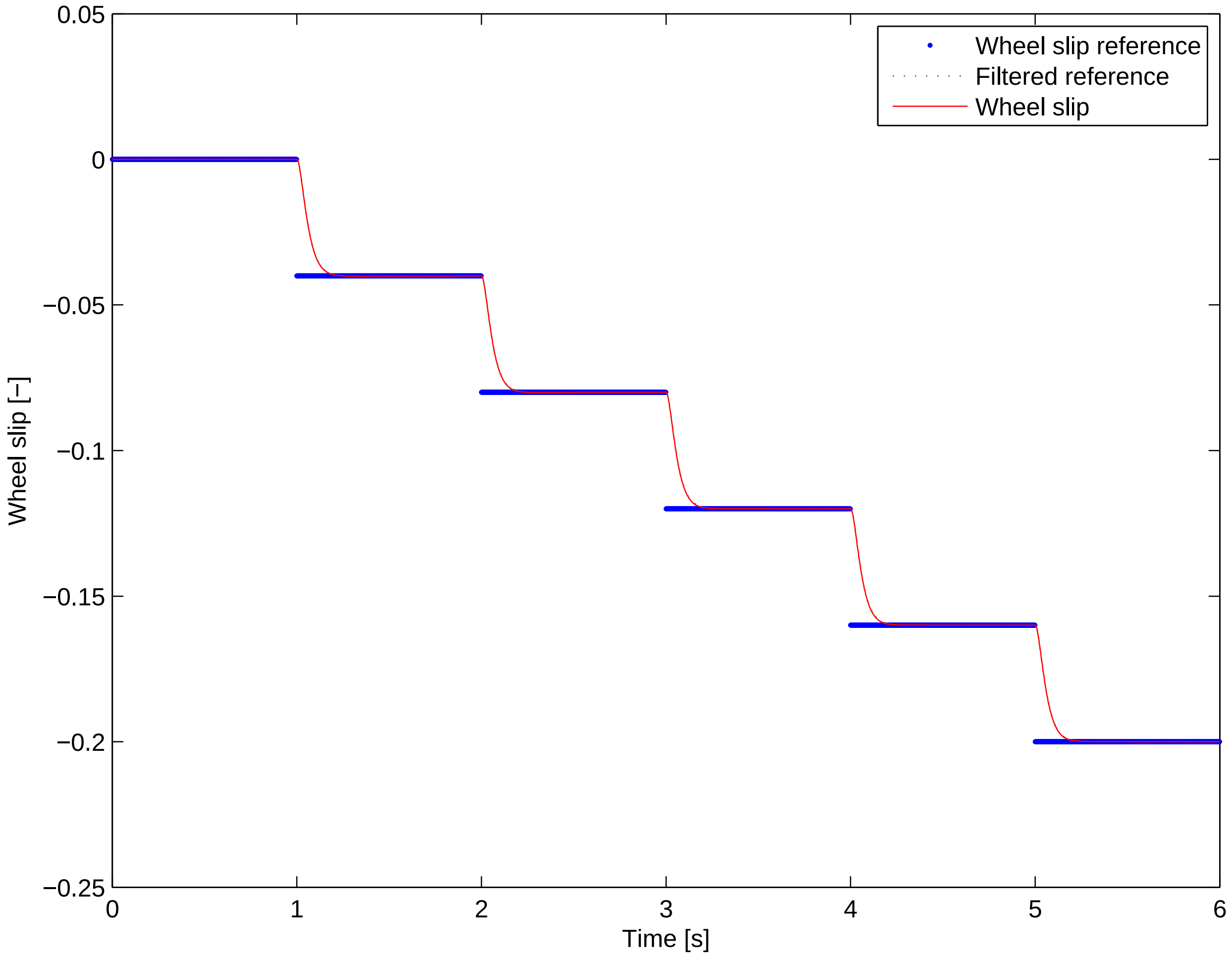


Simulations — Pure feedback control

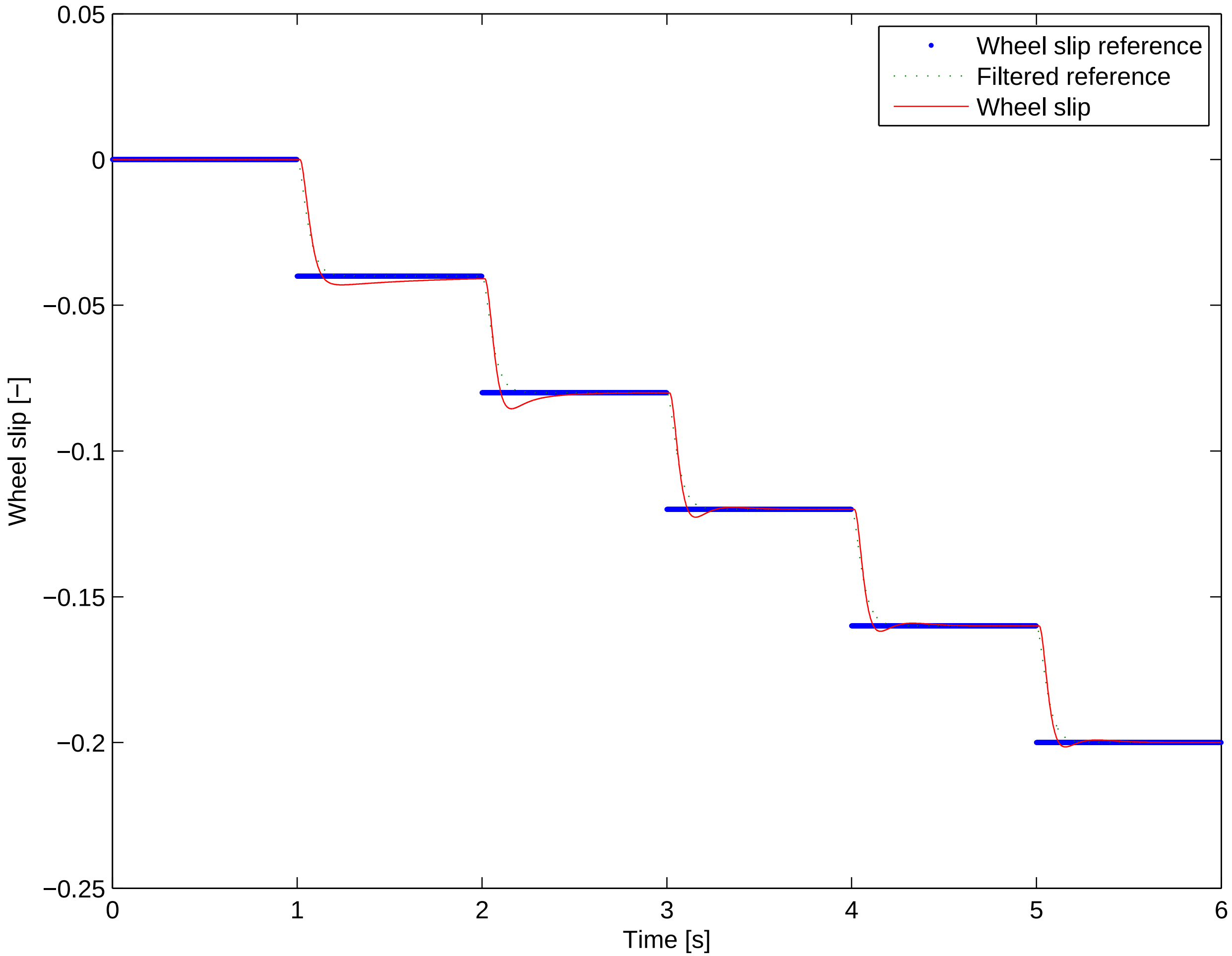


CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

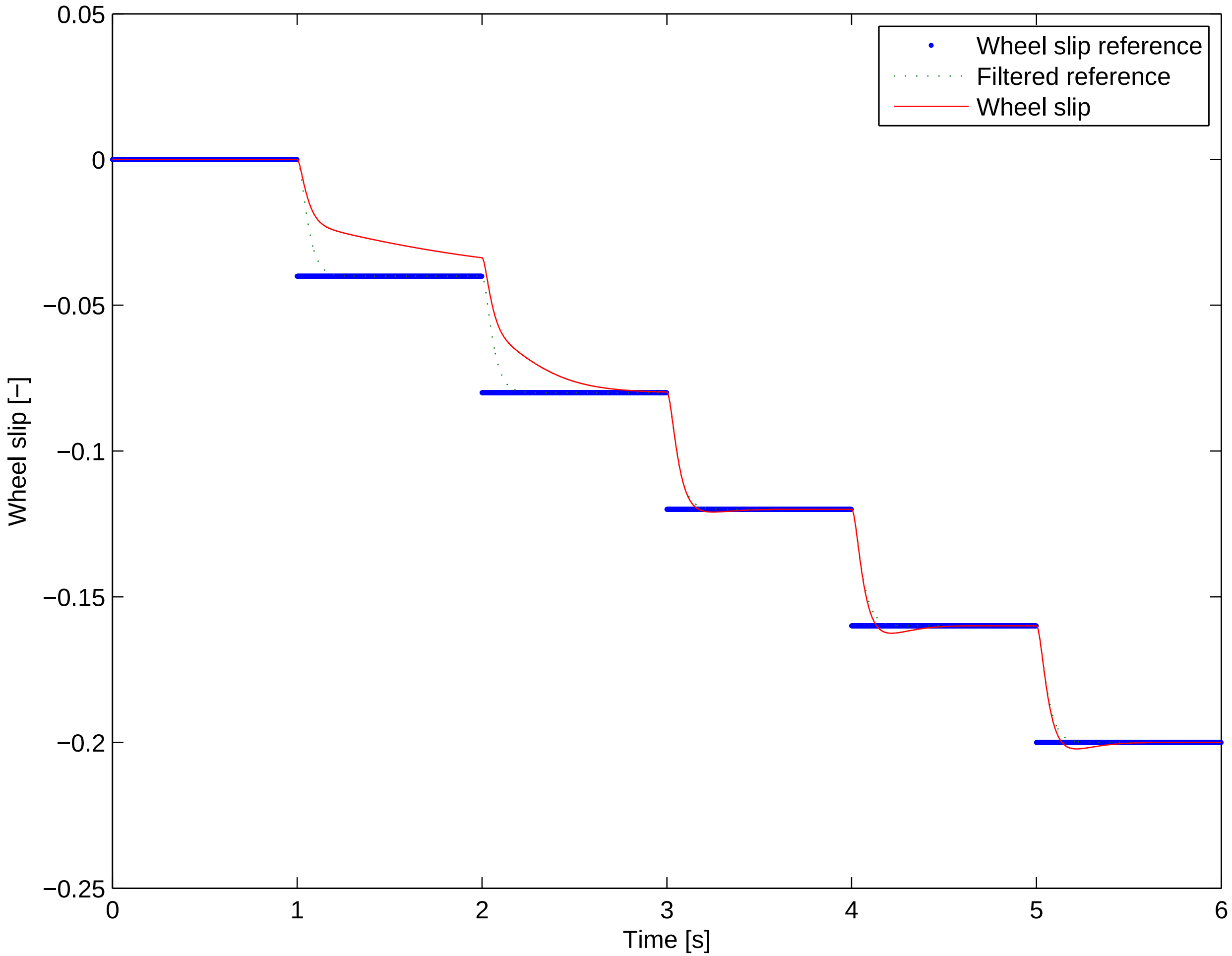
Simulations — With both feedback and feedforward control



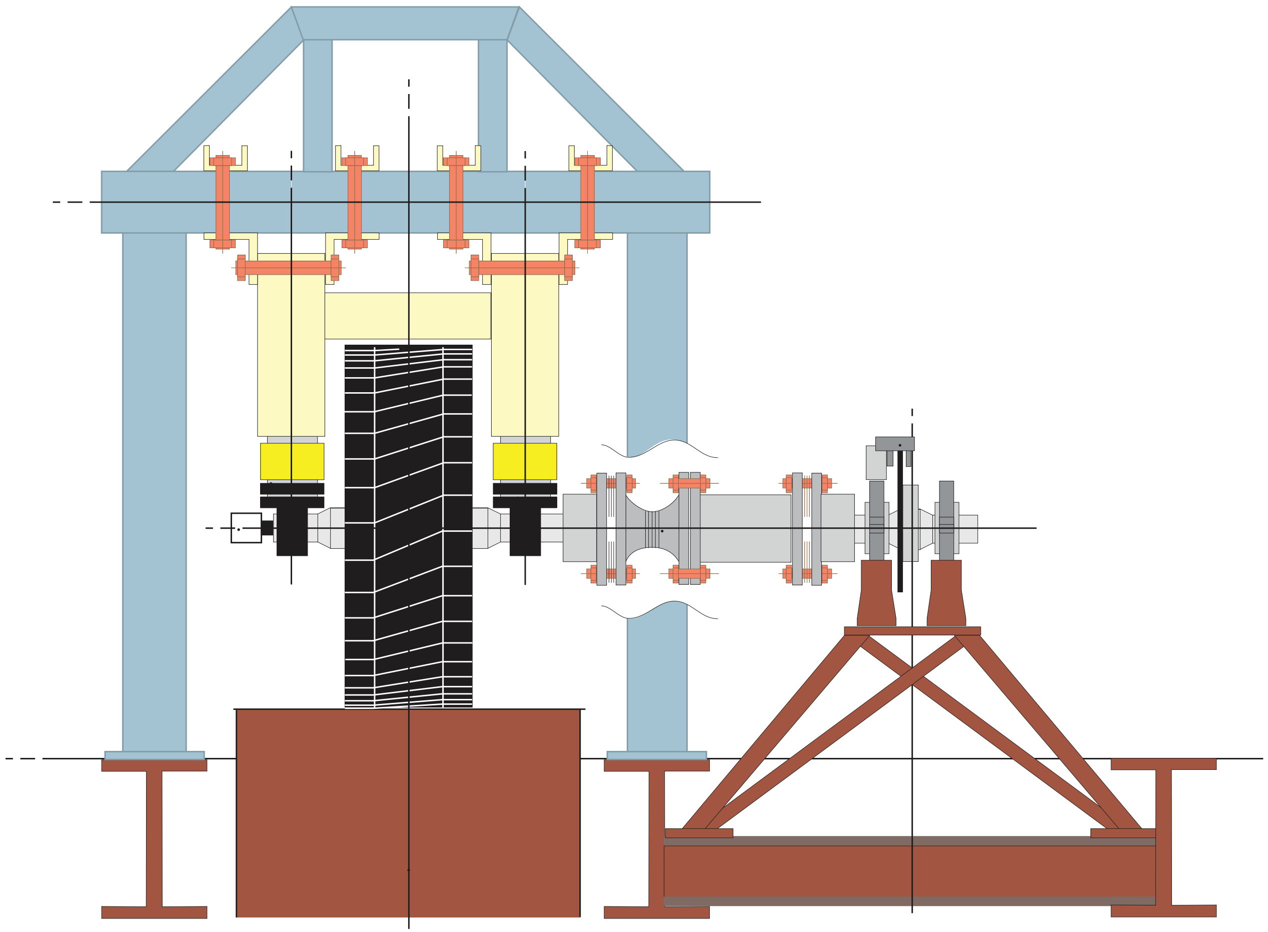
Simulations — With a delay of $15\,ms$



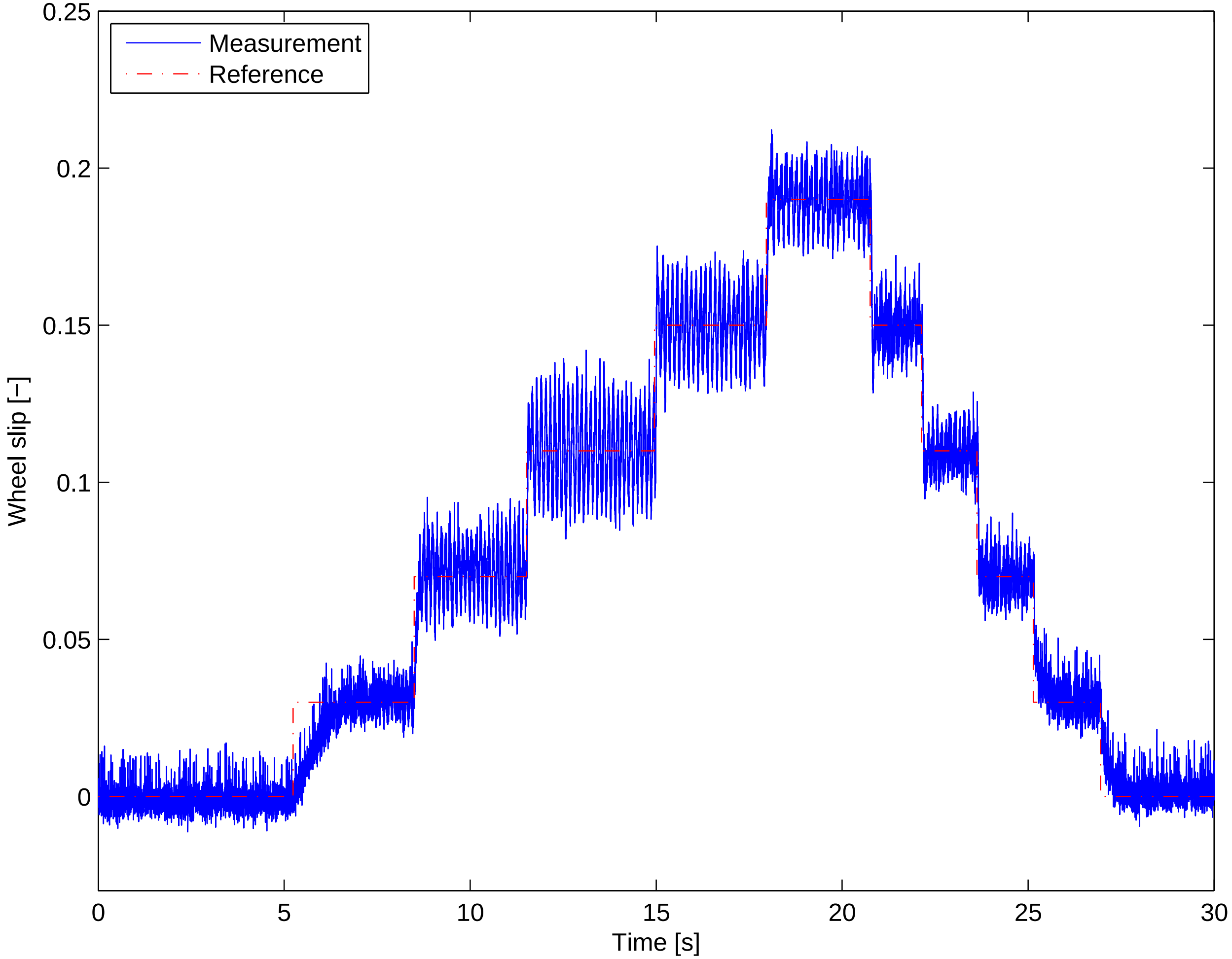
Simulations — With a perturbation of $\mu(\cdot)$



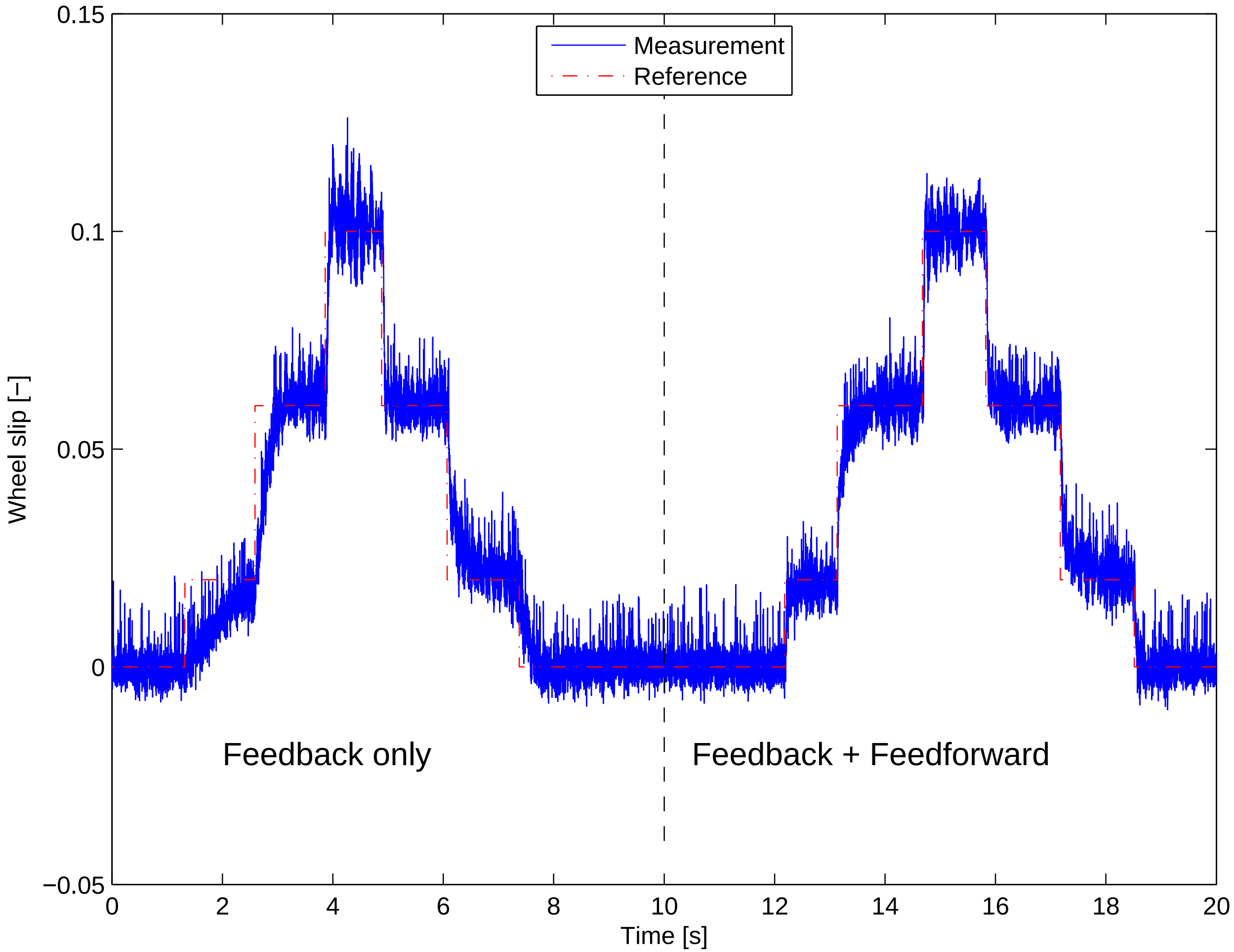
TU Delft's Tyre Setup



Experimental validation, with Mathieu Gerard (TU Delft)

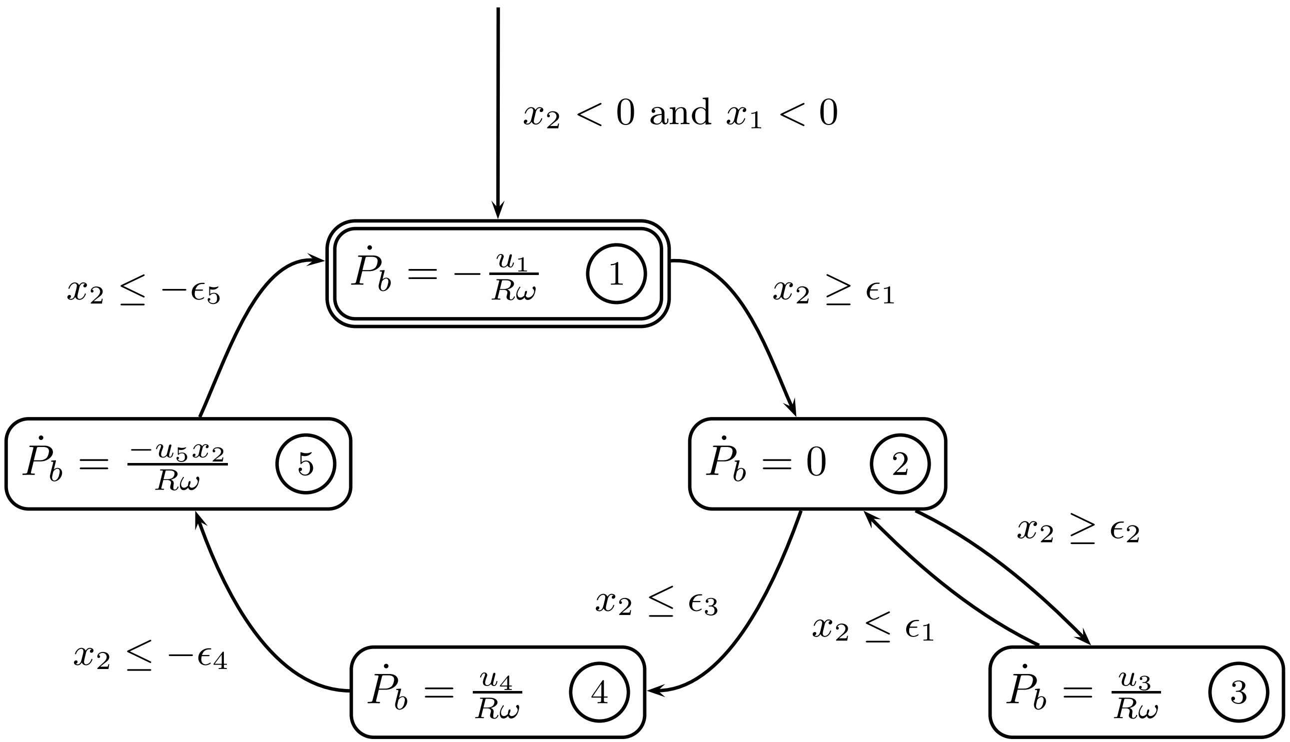


Experimental validation, with Mathieu Gerard (TU Delft)

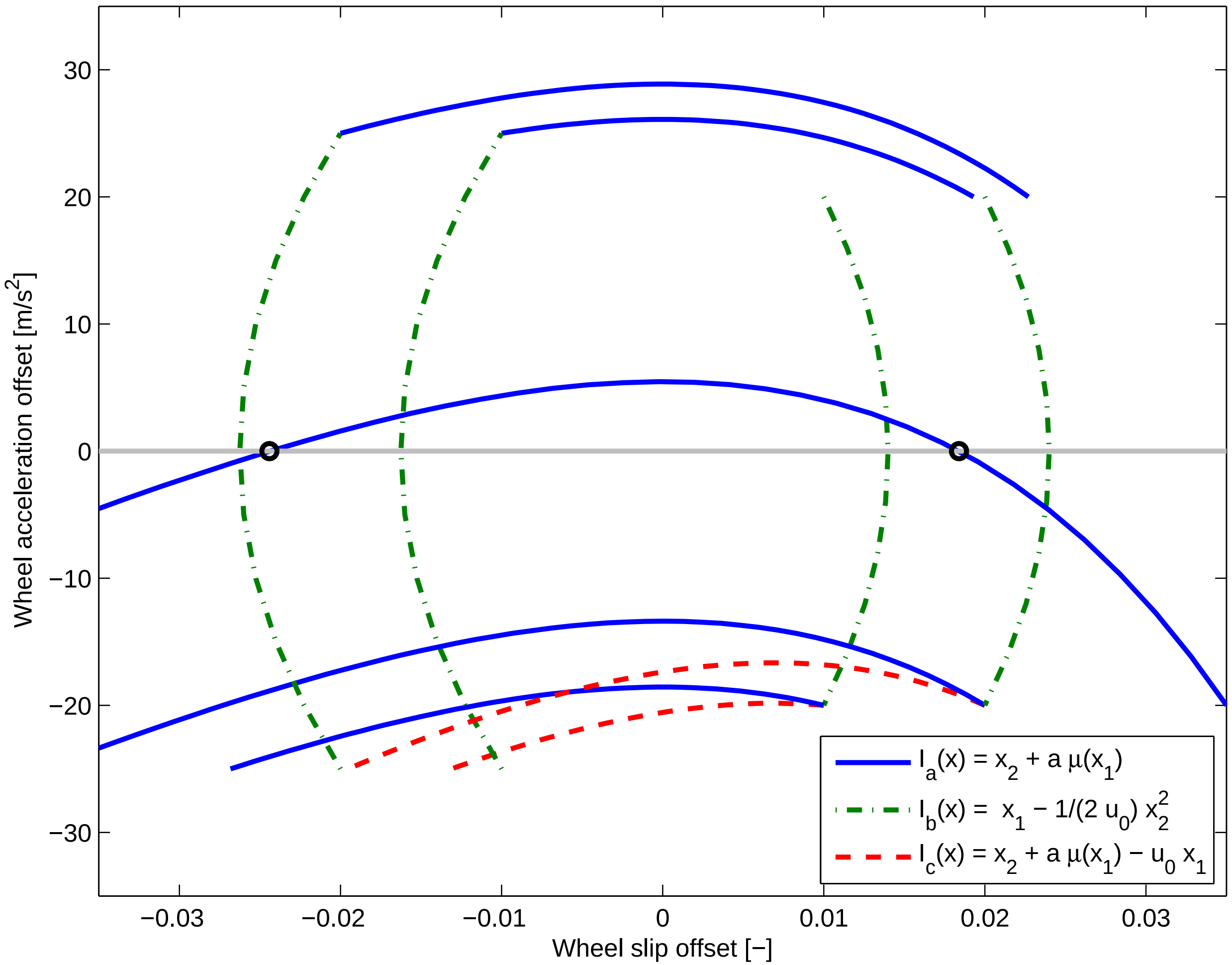


CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

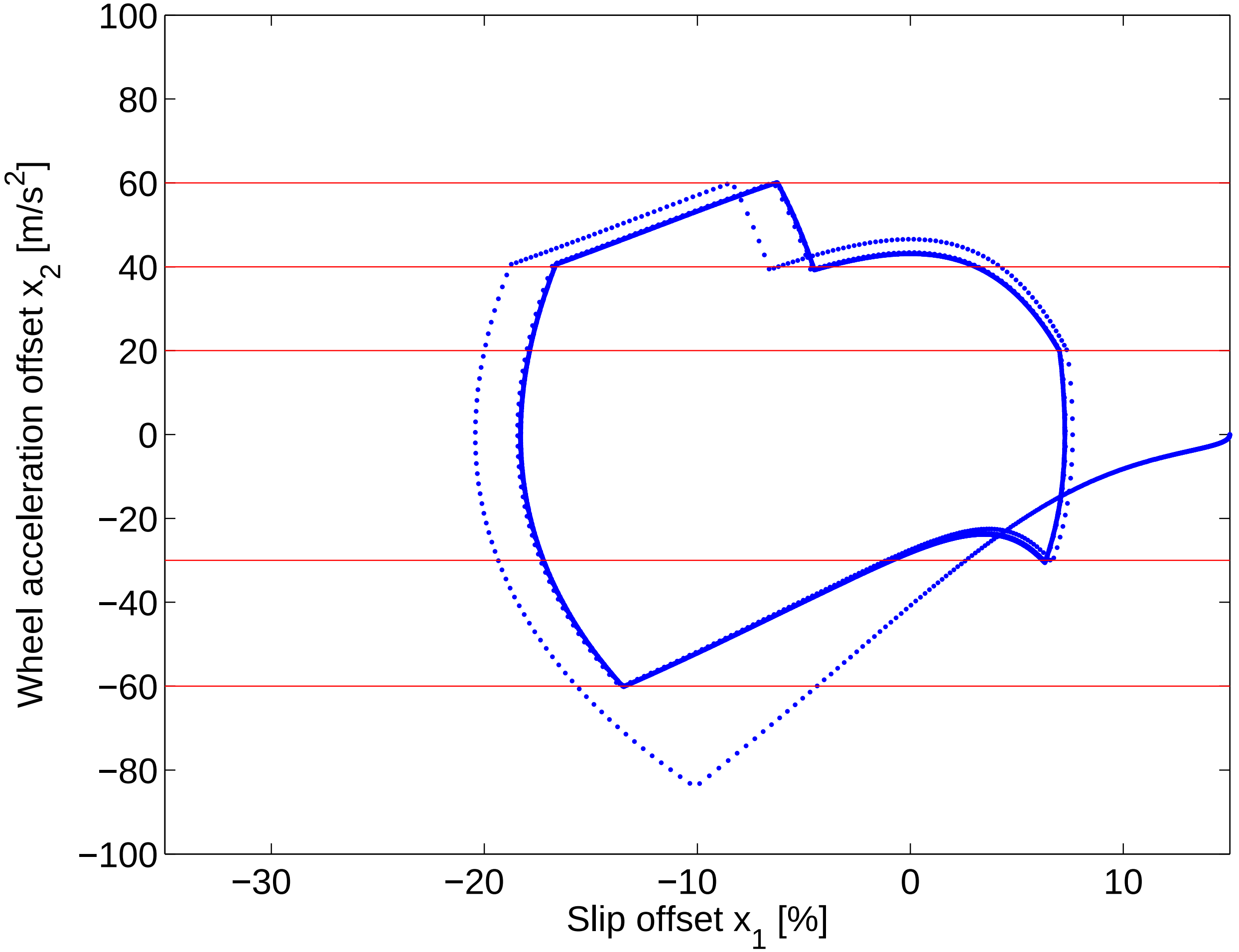
Hybrid five-phase algorithms (WPL – VSD 2006)



A method based on first integrals

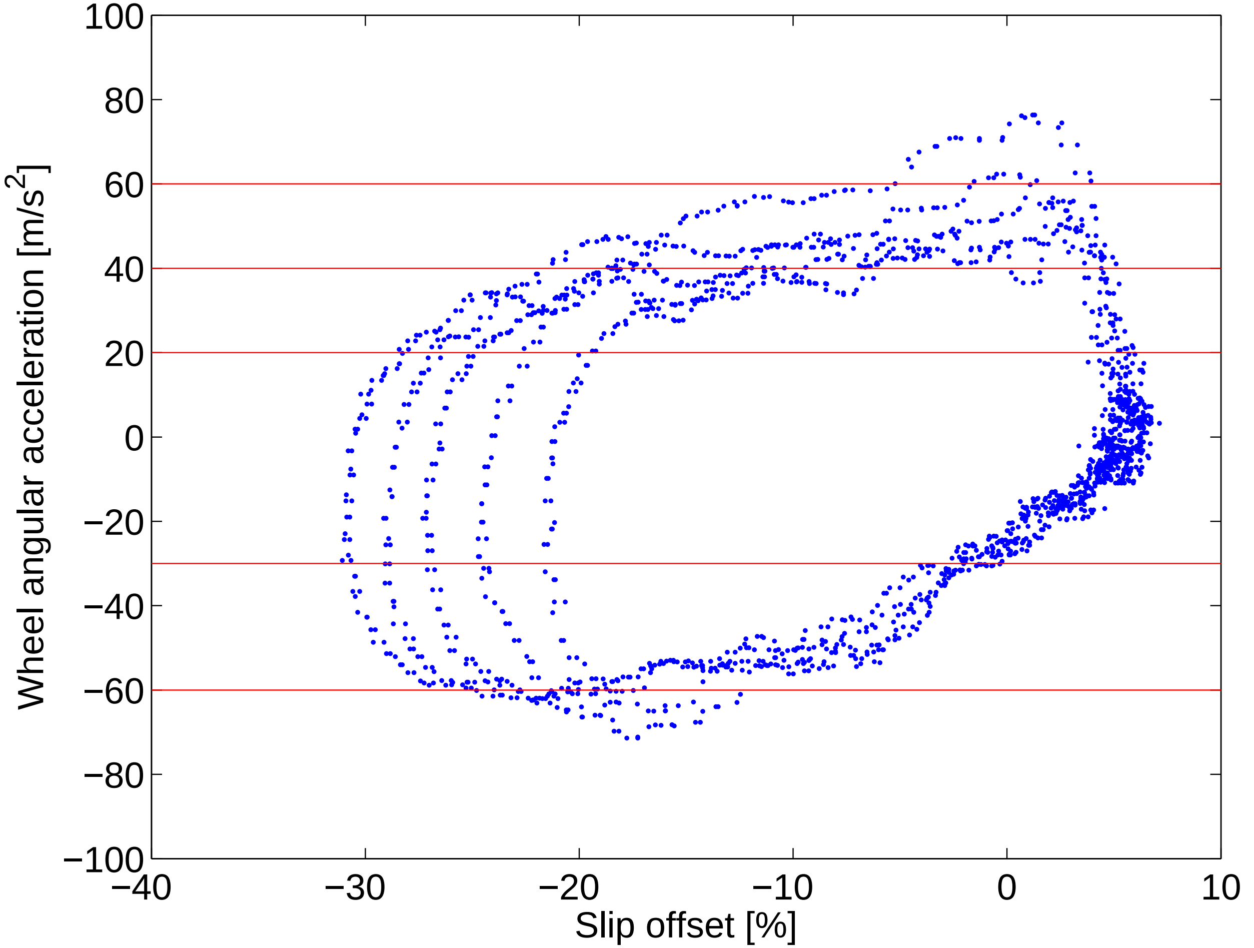


Simulation

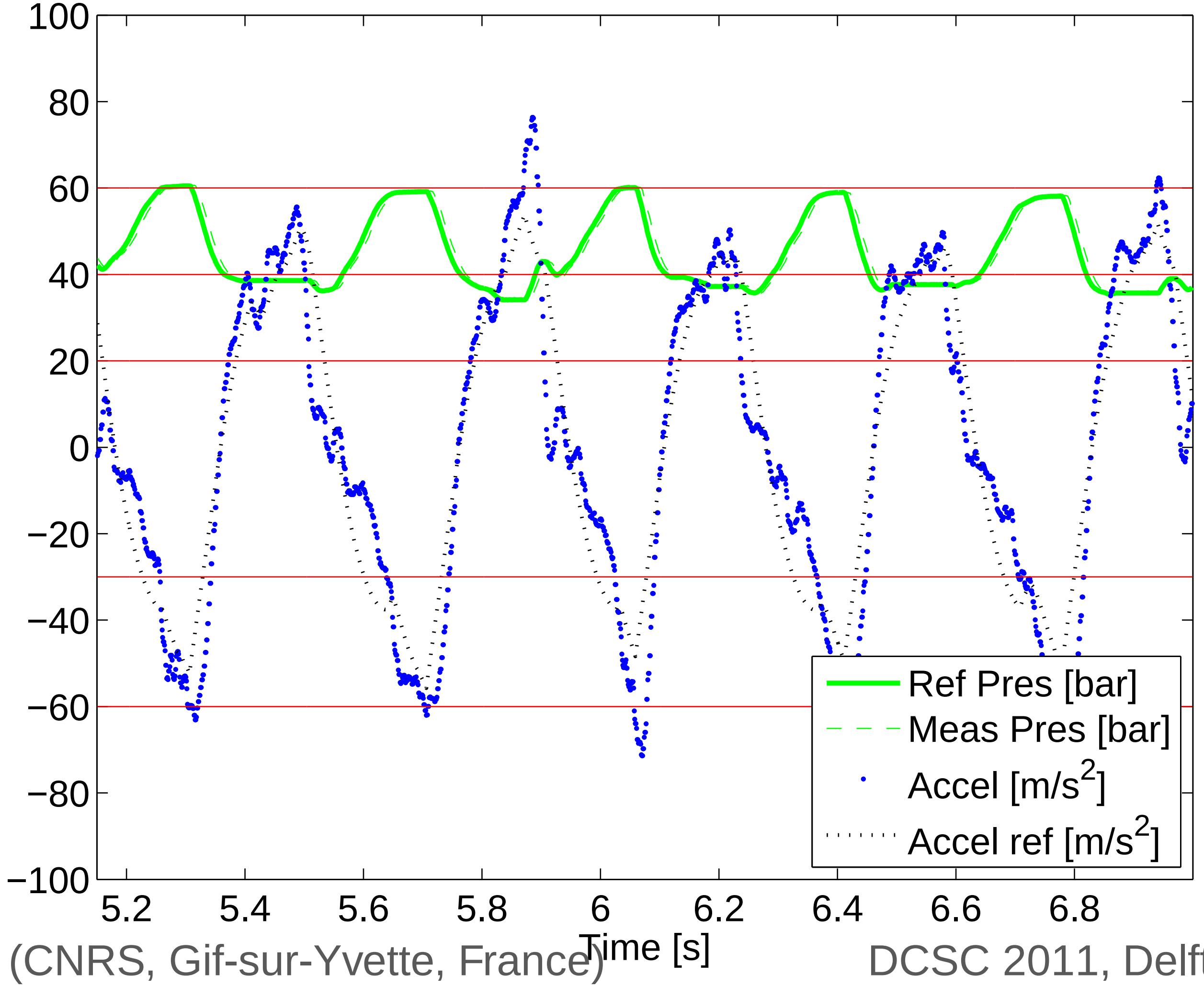

100
80
60
40
20
0
−20
−40
−60
−80
−100
Wheel acceleration offset x_2 [m/s^2]
−30
−20
−10
0
10
Slip offset x_1 [%]

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

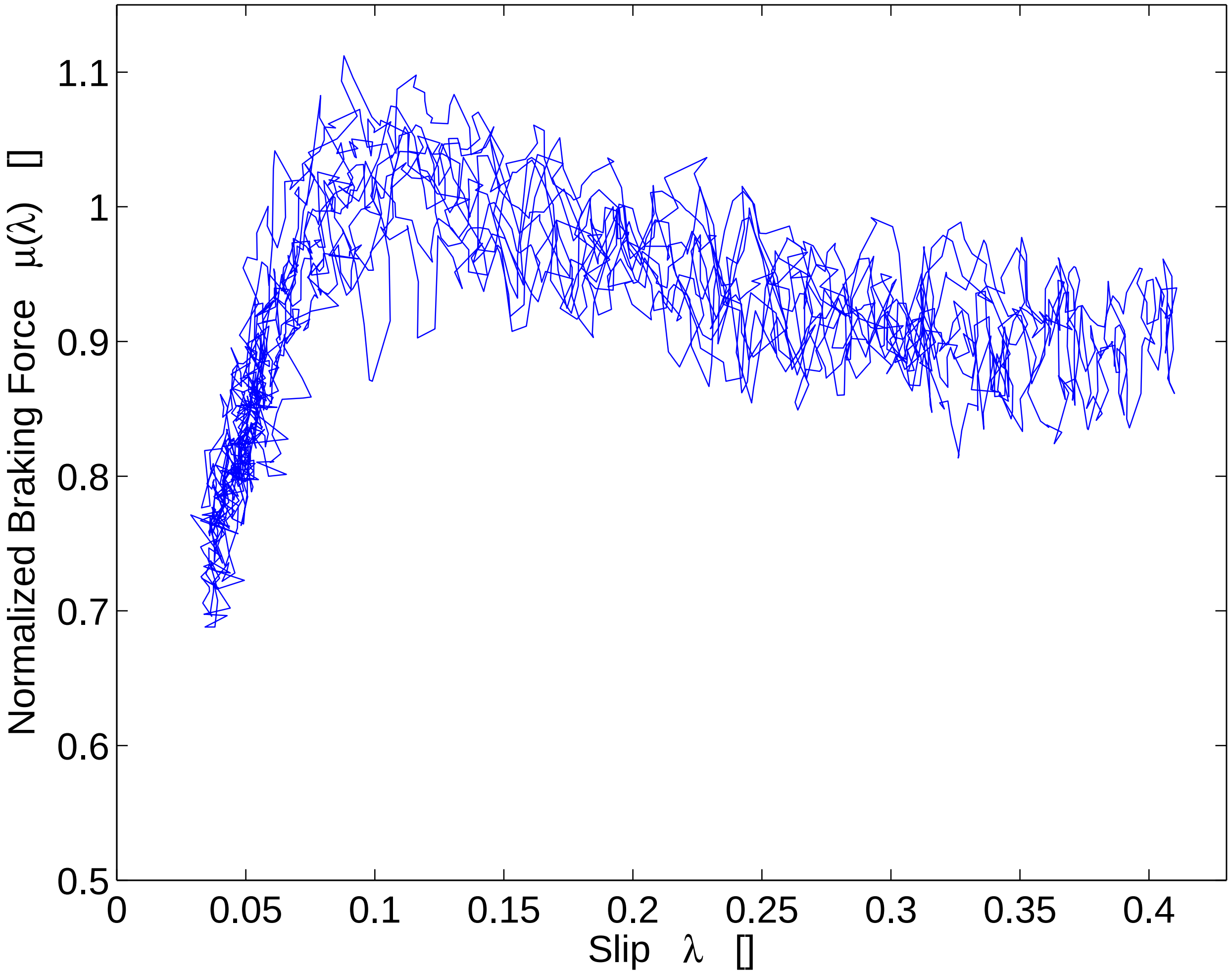
Experimental validation, with Mathieu Gerard (TU Delft)



Experimental validation, with Mathieu Gerard (TU Delft)



Experimental validation, with Mathieu Gerard (TU Delft)



Conclusion

- We proposed a new cascaded wheel slip controller.
- It uses a feedforward to speed up convergence, but a perfect knowledge of the tyre is not required (the feedback part does not use it).
- It leads to a proof of global exponential stability of the closed-loop system.
- Robust to practical phenomena (delays, relaxation length, tyre parameters).
- Validated experimentally with a tyre in-the-loop, by Mathieu Gerard (TU Delft).

Perspectives

- A controller that takes into account actuation delays is currently developed.
- The algorithms for computing angular wheel acceleration need to be improved.

Publications

[1] W. Pasillas-Lépine. Hybrid modelling and limit cycle analysis for a class of anti-lock brake algorithms. In *Proceedings of the Advanced Vehicle Control Congress*, Arnhem (Holland), 2004.

[2] I. Ait-Hammouda and W. Pasillas-Lépine. On a class of eleven-phase anti-lock brake algorithms robust with respect to discontinuous transitions of road characteristics. In *Proc. of the IFAC Symposium on Systems Structure and Control*, Oaxaca (Mexico), 2004.

[3] W. Pasillas-Lépine. Hybrid modelling and limit cycle analysis for a class of five-phase ABS algorithms. *Vehicle System Dynamics*, 44(2) :173–188, 2006.

[4] I. Ait-Hammouda and W. Pasillas-Lépine. Jumps and synchronization in anti-lock brake algorithms. In *Proc. of the Advanced Vehicle Control Congress*, Kobe (Japan), 2008.

[5] M. Gerard, W. Pasillas-Lépine, E. de Vries, and M. Verhaegen. Adaptation of hybrid five-phase ABS algorithms for experimental validation. In *Proc. of the IFAC Symposium on Advances in Automotive Control*, Munich (Germany), 2010.

[6] M. Gerard, A. Loría, W. Pasillas-Lépine, and M. Verhaegen. Experimental validation of a cascaded wheel slip control strategy. In *Proc. of the Advanced Vehicle Control Congress*, Loughborough (United-Kingdom), 2010.

[7] W. Pasillas-Lépine and A. Loría. A new mixed wheel slip and acceleration control based on a cascaded design. In *Proc. of the IFAC Symposium on Nonlinear Control Systems*, Bologna (Italy), 2010.

[8] M. Gerard, W. Pasillas-Lépine, E. de Vries, and M. Verhaegen. Improvements to a five-phase ABS algorithm for experimental validation. *Vehicle System Dynamics*, 50(10) :1585–1611, 2012.

[9] W. Pasillas-Lépine, A. Loría, and M. Gerard. Design and validation of a new mixed wheel slip and acceleration controller. *Automatica*, 48(8) :1852–1859, 2012.